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GENERAL LAWS GOVERNING IN MECHANICAL VIBRATORY SYSTEMS

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A method is proposed for investigating complex dynamic mechanisms with the use of a sequence of differential operators, which allows one to by pass cumbersome mathematical calculations, transform the system of three coupled differential equations of the second order into system of three independent linear inhomogeneous differential equations of the sixth order with constant coefficients, and represent the laws of influence of the physical parameters of a mechanical system on its dynamic properties in a simple analytical form. The indicated method can be used for investigating the vibrations of passive vibration dampers as well as their vibration resistance and quality and can be extended to other linear and linearized mechanical vibratory systems of higher orders.

Methods of analysis of the vibrations of mechanical systems have found wide use in modern machine building for designing vibration dampers, vibration isolators, and restraining arms used in overstressed mobile machines, aviation equipment, subway rolling stocks, trains, and machine tools. One of this methods is the method of chain dynamic systems or total mechanical resistance — impedance [1]. However, the indicated method is based on a simplified mathematical model of the dynamic and kinematic parameters of an actual object, which breaks the logic of dynamic processes and the Newtonian mechanics laws [2]. Another method of investigating vibratory systems, which most reliably accounts for their physical properties, is the method of amplitude-frequency characteristics [3]. The essence of this method is that the dynamics of a complex mechanical object is adequately defined by a system of linear differential equations solved using the integral Laplace transform, which makes it possible to analyze a mechanical system can be analyzed with the use of transfer functions and frequency characteristics and obtain the desired solutions using Riemann–Mellin formulas; however, this is frequently different to realize.

In the present work we propose a method of investigating vibratory systems defined by linear and linearized equations [4–7] with a large number of unknown variables, which allows one to transform coupled second-order differential equations into independent linear inhomogeneous differential equations, to solve them exactly, and to obtain a characteristic polynomial, and to determine the amplitude-frequency characteristics and transfer functions of a system and its vibration resistance, quality, and vibration-resistance margin.

As a vibratory system, we will consider a passive uniaxial vibration isolator used in mobile machines. The physical properties of the materials of passive dampers do not change with time unlike the properties of the materials of active dampers containing electrorheological or marnetorheological liquids. The isolator considered consists of metal springs, silent units, rubber-metal shock absorbers, and hydraulic supports. Such mechanisms are designed with the use of the simple elastic and damping elements, the properties of which depend linearly on their vibrations and the speed of these vibrations. The vibration isolator, the general dynamic diagram of which is presented in Fig. 1, can be defined by the system of four coupled inhomogeneous differential equations with indivisible variables

$$\ddot{x}_0 = -b_{00}\dot{x}_0 + b_{01}\dot{x}_1 + b_{02}\dot{x}_2 + b_{03}\dot{x}_3 - c_{00}x_0 + c_{01}x_1 + c_{02}x_2 + c_{03}x_3 + f\,, \tag{1}$$

$$\ddot{x}_1 = b_{10}\dot{x}_0 - b_{11}\dot{x}_1 + b_{12}\dot{x}_2 + b_{13}\dot{x}_3 + c_{10}x_0 - c_{11}x_1 + c_{12}x_2 + c_{13}x_3 + f_1\,, \tag{2}$$

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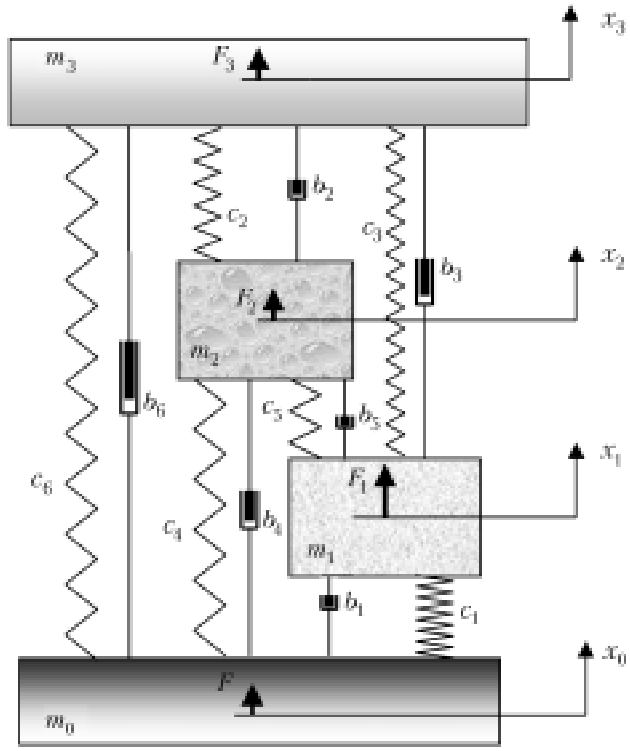

Fig. 1. General dynamic system of vibration isolation.

$$\ddot{x}_2 = b_{20}\dot{x}_0 + b_{21}\dot{x}_1 - b_{22}\dot{x}_2 + b_{23}\dot{x}_3 + c_{20}x_0 + c_{21}x_1 - c_{22}x_2 + c_{23}x_3 + f_2 , \tag{3}$$

$$\ddot{x}_3 = b_{30}\dot{x}_0 + b_{31}\dot{x}_1 + b_{32}\dot{x}_2 - b_{33}\dot{x}_3 + c_{30}x_0 + c_{31}x_1 + c_{32}x_2 - c_{33}x_3 + f_3 , \tag{4}$$

where $b_{00} = (b_1 + b_4 + b_6)/m_0$, $b_{01} = b_1/m_0$, $b_{02} = b_4/m_0$, $b_{03} = b_6/m_0$, $b_{10} = b_1/m_1$, $b_{11} = (b_1 + b_3 + b_5)/m_1$, $b_{12} = b_5/m_1$, $b_{13} = b_3/m_1$, $b_{20} = b_4/m_2$, $b_{21} = b_5/m_2$, $b_{22} = (b_2 + b_4 + b_5)/m_2$, $b_{23} = b_2/m_2$, $b_{30} = b_6/m_3$, $b_{31} = b_3/m_3$, $b_{32} = b_2/m_3$, $b_{33} = (b_2 + b_3 + b_6)/m_3$, $c_{00} = (c_1 + c_4 + c_6)/m_0$, $c_{01} = c_1/m_0$, $c_{02} = c_4/m_0$, $c_{03} = c_6/m_0$, $c_{10} = c_1/m_1$, $c_{11} = (c_1 + c_3 + c_5)/m_1$, $c_{12} = c_5/m_1$, $c_{13} = c_3/m_1$, $c_{20} = c_4/m_2$, $c_{21} = c_5/m_2$, $c_{22} = (c_2 + c_4 + c_5)/m_2$, $c_{23} = c_2/m_2$, $c_{30} = c_6/m_3$, $c_{31} = c_3/m_3$, $c_{32} = c_2/m_3$, $c_{33} = (c_2 + c_3 + c_6)/m_3$, $f(t) = a \sin \omega t$, $a = A/m_0$, $f_1 = F_1/m_1$, $f_2 = F_2/m_2$, $f_3 = F_3/m_3$, and $F_i = 0$ for all $i = \overline{1,\ 3}$. In the vibroisolation system presented in Fig. 1, m_0 represents a rigid base with a mass exceeding, in the physical equivalent, the other parameters: $m_0 \to \infty$; in this case, $\ddot{x} = a \sin \omega t$, $\dot{x}_0 = -(a/\omega) \cos \omega t$, and $x_0 = -(a/\omega^2) \sin \omega t$. In this connection the mathematical model of mechanical vibrations (1)–(4) becomes simpler:

$$\ddot{x}_1 = -b_{11}\dot{x}_1 + b_{12}\dot{x}_2 + b_{13}\dot{x}_3 - c_{11}x_1 + c_{12}x_2 + c_{13}x_3 + \mathcal{F}_1 , \tag{5}$$

$$\ddot{x}_2 = b_{21}\dot{x}_1 - b_{22}\dot{x}_2 + b_{23}\dot{x}_3 + c_{21}x_1 - c_{22}x_2 + c_{23}x_3 + \mathcal{F}_2 , \tag{6}$$

$$\ddot{x}_3 = b_{31}\dot{x}_1 + b_{32}\dot{x}_2 - b_{33}\dot{x}_3 + c_{31}x_1 + c_{32}x_2 - c_{33}x_3 + \mathcal{F}_3 , \tag{7}$$

where $\mathcal{F}_1 = f_1 - (a/\omega^2)c_{10} \sin \omega t - (a/\omega)b_{10} \cos \omega t$, $\mathcal{F}_2 = f_2 - (a/\omega^2)c_{20} \sin \omega t - (a/\omega)b_{20} \cos \omega t$, and $\mathcal{F}_3 = f_3 - (a/\omega^2)c_{30} \sin \omega t - (a/\omega)b_{30} \cos \omega t$.

Equations of this type are frequently used in mechanics for determining the vibrations of a ship, a ship gyroscope, frictionally coupled vibratory systems, a horizontal pendulum, and vibration-isolated objects. It is impossible to integrate the indicated system of differential equations in the general form. It can be solved in the case where the right sides of these equations are specially selected and the coefficients b_{ij} and c_{ij} satisfy certain conditions (i = 1, 2; j = 1, 2) [4]. Modern computer means make it possible to obtain numerical results only for small time intervals and concrete parameters of the problem, by which the influence of the coefficients on the total vibrational process cannot be judged. The discrepancy of the system of differential equations (5)–(7) results in errors appearing and the calculations are terminated for a limiting number of steps. A problem of system (5)–(7) is the connection of variables and of their first and second derivatives. Let us separate the derivatives, using the operation of repeated differentiation, which is not contrary to the Peano existence and uniqueness theorem [4]. For this purpose, we will separate the following differential operators of the second and first orders from the system of equations (5)–(7):

$$L_1 = \frac{d^2}{dt^2} + b_{11}\frac{d}{dt} + c_{11}\,, \quad L_2 = \frac{d^2}{dt^2} + b_{22}\frac{d}{dt} + c_{22}\,, \quad L_3 = \frac{d^2}{dt^2} + b_{33}\frac{d}{dt} + c_{33}\,, \tag{8}$$

$$d_{12} = b_{12}\frac{d}{dt} + c_{12}\,, \quad d_{13} = b_{13}\frac{d}{dt} + c_{13}\,, \quad d_{21} = b_{21}\frac{d}{dt} + c_{21}\,,$$

$$d_{23} = b_{23}\frac{d}{dt} + c_{23}\,, \quad d_{31} = b_{31}\frac{d}{dt} + c_{31}\,, \; d_{32} = b_{32}\frac{d}{dt} + c_{32}\,. \tag{9}$$

The dynamic system of vibration isolation (5)–(7) can be represented in the form

$$L_1\,(x_1) = d_{12}\,(x_2) + d_{13}\,(x_3) + \mathcal{F}_1\,, \tag{10}$$

$$L_2\,(x_2) = d_{21}\,(x_1) + d_{23}\,(x_3) + \mathcal{F}_2\,, \tag{11}$$

$$L_3\,(x_3) = d_{31}\,(x_1) + d_{32}\,(x_2) + \mathcal{F}_3\,. \tag{12}$$

We successively applied the operators L_i ($i = \overline{1, 3}$) to the differential equations (10)–(12) in relation to the variables x_i and obtained a new system of three independent sixth-order differential equations. The left sides of these equations have identical coefficients because of the invariance of the differential operators (8) and (9) used:

$$D^6\,(x_i) = L_1L_2L_3\,(x_i) - L_1d_{23}d_{32}\,(x_i) - L_2d_{13}d_{31}\,(x_i) -$$

$$- L_3d_{12}d_{21}\,(x_i) - d_{12}d_{23}d_{31}\,(x_i) - d_{13}d_{32}d_{21}\,(x_i)\,. \tag{13}$$

This points to the fact that the mechanical vibratory system has set of physical parameters characteristic of only it. The characteristic equations are identical to the characteristic polynomial obtained by the method of integral Laplace transform. The natural vibrations differ by the numerical parameters of their initial conditions.

The right sides of the new system of differential equations are different and characterize the forced vibrations of a nonconservative mechanical system

$$D^6\,(x_i) = L_jL_k\,(\mathcal{F}_i) - d_{jk}d_{kj}\,(\mathcal{F}_i) + L_jd_{ik}\,(\mathcal{F}_k) + d_{ij}d_{jk}\,(\mathcal{F}_k) + L_kd_{ij}\,(\mathcal{F}_j) +$$

$$+ d_{ik}d_{kj}\,(\mathcal{F}_j)\,, \quad i \neq j \neq k\,, \quad i = \overline{1, 3}\,, \quad j = \overline{1, 3}\,, \quad k = \overline{1, 3}\,. \tag{14}$$

Let us write the general system of three independent differential equations of the sixth order in the operator form

$$- d_{13}d_{32}d_{21}\,(x_i) = L_jL_k\,(\mathcal{F}_i) - d_{jk}d_{kj}\,(\mathcal{F}_i) + L_jd_{ik}\,(\mathcal{F}_k) + d_{ij}d_{jk}\,(\mathcal{F}_k) +$$

$$+ L_kd_{jj}\,(\mathcal{F}_j) + d_{jk}d_{kj}\,(\mathcal{F}_j)\,, \quad i = \overline{1, 3}\,, \quad j = \overline{1, 3}\,, \quad k = \overline{1, 3}\,, \quad i \neq j \neq k\,. \tag{15}$$

If $i = 1$, $j = 2$ and $k = 3$, if $i = 2$, $j = 3$ and $k = 1$, and if $i = 3$, $j = 1$ and $k = 2$. This is a Boole permutation.

The system of equations (15) allows one to determine, in explicit form, the constant coefficients of the linear inhomogeneous sixth-order differential equations:

$$x_i^{\mathrm{VI}} + \Delta_5 x_i^{\mathrm{V}} + \Delta_4 x_i^{\mathrm{IV}} + \Delta_3 x_i^{\mathrm{III}} + \Delta_2 x_i'' + \Delta_1 x_i' + \Delta_0 x_i =$$

$$= \mathcal{F}_i^{\mathrm{IV}} + (b_{jj} + b_{kk})\, \mathcal{F}_i^{\mathrm{III}} + b_{ij}\mathcal{F}_j^{\mathrm{III}} + b_{ik}\mathcal{F}_k^{\mathrm{III}} + \Delta^2_{\mathcal{F}_i}\mathcal{F}_i^{\mathrm{II}} +$$

$$+ \Delta^2_{\mathcal{F}_j}\mathcal{F}_j^{\mathrm{II}} + \Delta^2_{\mathcal{F}_k}\mathcal{F}_k^{\mathrm{II}} + \Delta^{\mathrm{I}}_{\mathcal{F}_i}\mathcal{F}_i^{\mathrm{I}} + \Delta^{\mathrm{I}}_{\mathcal{F}_j}\mathcal{F}_j^{\mathrm{I}} + \Delta^{\mathrm{I}}_{\mathcal{F}_k}\mathcal{F}_k^{\mathrm{I}} + \Delta_{\mathcal{F}_i}, \quad i = \overline{1,3}\,. \tag{16}$$

Here

$$\Delta_0 = \begin{vmatrix} c_{11} & -c_{12} & c_{13} \\ -c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}, \quad \Delta_1 = \begin{vmatrix} b_{11} & -c_{12} & c_{13} \\ -b_{21} & c_{22} & c_{23} \\ b_{31} & c_{32} & c_{33} \end{vmatrix} + \begin{vmatrix} c_{11} & -b_{12} & c_{13} \\ -c_{21} & b_{22} & c_{23} \\ c_{31} & b_{32} & c_{33} \end{vmatrix} + \begin{vmatrix} c_{11} & -c_{12} & b_{13} \\ -c_{21} & c_{22} & b_{23} \\ c_{31} & c_{32} & b_{33} \end{vmatrix},$$

$$\Delta_2 = \begin{vmatrix} b_{11} & -b_{12} & c_{13} \\ -b_{21} & b_{22} & c_{23} \\ b_{31} & b_{32} & c_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & -c_{12} & b_{13} \\ -b_{21} & c_{22} & b_{23} \\ b_{31} & c_{32} & b_{33} \end{vmatrix} + \begin{vmatrix} c_{11} & -b_{12} & b_{13} \\ -c_{21} & b_{22} & b_{23} \\ c_{31} & c_{32} & b_{33} \end{vmatrix} + \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} + \begin{vmatrix} c_{11} & c_{13} \\ c_{31} & c_{33} \end{vmatrix} + \begin{vmatrix} c_{22} & c_{23} \\ c_{32} & c_{33} \end{vmatrix},$$

$$\Delta_3 = \begin{vmatrix} b_{11} & b_{12} & -b_{13} \\ b_{21} & b_{22} & b_{23} \\ -b_{31} & b_{32} & b_{33} \end{vmatrix} + \begin{vmatrix} c_{11} & c_{12} \\ b_{21} & b_{22} \end{vmatrix} + \begin{vmatrix} c_{11} & c_{13} \\ b_{31} & b_{33} \end{vmatrix} + \begin{vmatrix} b_{22} & c_{23} \\ b_{32} & c_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{13} \\ c_{31} & c_{33} \end{vmatrix} + \begin{vmatrix} c_{22} & b_{23} \\ c_{32} & b_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{12} \\ c_{21} & c_{22} \end{vmatrix},$$

$$\Delta_4 = c_{11} + c_{22} + c_{33} + \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{13} \\ b_{31} & b_{33} \end{vmatrix} + \begin{vmatrix} b_{22} & b_{23} \\ b_{32} & b_{33} \end{vmatrix}, \quad \Delta_5 = b_{11} + b_{22} + b_{33},$$

$$\Delta^2_{\mathcal{F}_i} = c_{jj} + c_{kk} + \begin{vmatrix} b_{jj} & b_{jk} \\ b_{kj} & b_{kk} \end{vmatrix}, \quad \Delta^1_{\mathcal{F}_i} = \begin{vmatrix} b_{jj} & c_{jk} \\ b_{kj} & c_{kk} \end{vmatrix} + \begin{vmatrix} c_{jj} & b_{jk} \\ c_{kj} & b_{kk} \end{vmatrix},$$

$$\Delta^2_{\mathcal{F}_j} = c_{ij} + \begin{vmatrix} b_{ij} & -b_{ik} \\ b_{kj} & b_{kk} \end{vmatrix}, \quad \Delta^1_{\mathcal{F}_j} = \begin{vmatrix} b_{ij} & -c_{ik} \\ b_{kj} & c_{kk} \end{vmatrix} + \begin{vmatrix} c_{ij} & -b_{ik} \\ c_{kj} & b_{kk} \end{vmatrix}, \quad \Delta^2_{\mathcal{F}_k} = c_{ik} + \begin{vmatrix} b_{ij} & -b_{ik} \\ b_{jj} & b_{jk} \end{vmatrix},$$

$$\Delta^1_{\mathcal{F}_k} = \begin{vmatrix} b_{ij} & -c_{ik} \\ b_{jj} & c_{jk} \end{vmatrix} + \begin{vmatrix} c_{ij} & -b_{ik} \\ c_{jj} & b_{jk} \end{vmatrix},$$

$$\Delta_{\mathcal{F}_1} = \begin{vmatrix} \mathcal{F}_1 & c_{12} & -c_{13} \\ -\mathcal{F}_2 & c_{22} & c_{23} \\ \mathcal{F}_3 & c_{32} & c_{33} \end{vmatrix}, \quad \Delta_{\mathcal{F}_2} = \begin{vmatrix} c_{11} & -\mathcal{F}_1 & c_{13} \\ c_{21} & \mathcal{F}_2 & -c_{23} \\ c_{31} & \mathcal{F}_3 & c_{33} \end{vmatrix}, \quad \Delta_{\mathcal{F}_3} = \begin{vmatrix} c_{11} & c_{12} & -\mathcal{F}_1 \\ c_{21} & c_{22} & \mathcal{F}_2 \\ c_{31} & -c_{32} & \mathcal{F}_3 \end{vmatrix}.$$

The general dynamic system of vibration isolation, presented in Fig. 1 and described by the differential equations (16), can be easily analyzed. For this purpose, it is necessary to determine the dynamic transfer functions; the frequency transfer functions; the amplitude-frequency characteristics; the resonance frequencies; the coefficients of dynamics, dynamic rigidity, and dynamic compliance; the stability of vibrations in the mechanism by the Routh, Hurwitz, Nyquist, and Mikhailov conditions; and adjust it to an optimum operation. For this purpose, we will use the integral Laplace transform with a complex parameter p at zero initial conditions. The functions $X_i(p)$ and $Y(p)$, $i = \overline{1,3}$, are Laplace representations of the originals $x_i(t)$ and $f(t)$. In this case, the dynamic system (16) will take the form

$$(p^6 + \Delta_5 p^5 + \Delta_4 p^4 + \Delta_3 p^3 + \Delta_2 p^2 + \Delta_1 p + \Delta_0)\, X_i(p) = (r_{5i}p^5 + r_{4i}p^4 + r_{3i}p^3 + r_{2i}p^2 + r_{1i}p + r_{0i})\, Y(p) +$$

$$+ d_{3i}p^3 + d_{2i}p^2 + d_{1i}p + d_{0i}\,, \quad i = \overline{1,3}\,. \tag{17}$$

The coefficients r_{mi} and d_{ni} ($m = \overline{0,5}$, $n = \overline{0,3}$) are constant quantities determined by the constants c_{ij}, b_{ij}, a, and ω, the values of which are not present because of the awkwardness of the corresponding expressions, and the coefficient d_{ni} is determined by the initial conditions as well as the function $f(t)$ and its derivatives. Let us assume that $f_1 = f_2 = f_3 = 0$ and $Y(p) = a\omega/(p^2 + \omega^2)$. System (5)–(7) is solved using the Riemann–Mellin formula of transformation of the Laplace transform into the system of equations

$$X_i(p) = \frac{r_{5i}p^5 + r_{4i}p^4 + r_{3i}p^3 + r_{2i}p^2 + r_{1i}p + r_{0i}}{p^6 + \Delta_5 p^5 + \Delta_4 p^4 + \Delta_3 p^3 + \Delta_2 p^2 + \Delta_1 p + \Delta_0}\, Y(p) +$$

$$+ \frac{d_{3i}p^3 + d_{2i}p^2 + d_{1i}p + d_{0i}}{p^6 + \Delta_5 p^5 + \Delta_4 p^4 + \Delta_3 p^3 + \Delta_2 p^2 + \Delta_1 p + \Delta_0}\,, \quad i = \overline{1,3}\,. \tag{18}$$

The first terms in (18) characterize the forced vibrations of the system and the second terms characterize its natural vibrations.

The dynamic transfer functions $W_i(p)$ are determined from Eqs. (17) [2]:

$$W_i(p) = \frac{r_{5i}p^5 + r_{4i}p^4 + r_{3i}p^3 + r_{2i}p^2 + r_{1i}p + r_{0i}}{p^6 + \Delta_5 p^5 + \Delta_4 p^4 + \Delta_3 p^3 + \Delta_2 p^2 + \Delta_1 p + \Delta_0} +$$

$$+ \frac{d_{3i}p^3 + d_{2i}p^2 + d_{1i}p + d_{0i}}{(p^6 + \Delta_5 p^5 + \Delta_4 p^4 + \Delta_3 p^3 + \Delta_2 p^2 + \Delta_1 p + \Delta_0)\, Y(p)}\,, \quad i = \overline{1,3}\,. \tag{19}$$

It is most interesting to investigate the influence of the polynomials of the first terms in formulas (19) on the vibration resistance and quality of the mechanical system as a whole. However, the most important information is contained in the coefficients of the characteristic polynomial of the denominator [5]. The second terms turn to zero when the variable $p = I\omega$ ($I = \sqrt{-1}$) in the expression $W_i(I\omega)$ for analysis of the amplitude-frequency characteristics of vibrations

$$\left|X_i(I\omega)\right| = a\sqrt{\frac{r_{4i}^2(\omega^2 - \omega_{*1i}^2)^2(\omega^2 - \omega_{*2i}^2)^2 + r_{5i}^2\omega^2(\omega^2 - \omega_{*3i}^2)^2(\omega^2 - \omega_{*4i}^2)^2}{(\omega^2 - \omega_{r1}^2)^2(\omega^2 - \omega_{r2}^2)^2(\omega^2 - \omega_{r3}^2)^2 + \Delta_5^2\omega^2(\omega^2 - \omega_{r4}^2)^2(\omega^2 - \omega_{r5}^2)^2}}\,, \quad i = \overline{1,3}\,. \tag{20}$$

Here ω_{*1i}, ω_{*2i}, ω_{*3i}, and ω_{*4i} are antiresonance frequencies and ω_{r1}, ω_{r2}, ω_{r3}, ω_{r4}, and ω_{r5} are resonance frequencies; in this case, ω_{*1i}, and ω_{*2i} are the roots of the polynomial $r_{4i}\omega^2 - r_{2i}\omega^2 r_{0i} = r_{4i}(\omega^2 - \omega_{*1i}^2)(\omega^2 - \omega_{*2i}^2)$, $r_{5i}\omega^5 - r_{3i}\omega^3 + r_{1i}\omega = r_{5i}\omega(\omega^2 - \omega_{*3i}^2)(\omega^2 - \omega_{*4i}^2)$, $-\omega^6 + \Delta_4\omega^4 - \Delta_2\omega^2 + \Delta_0 = -(\omega^2 - \omega_{r1}^2)(\omega^2 - \omega_{r2}^2)(\omega^2 - \omega_{r3}^2)$, $\Delta_5\omega^5 - \Delta_3\omega^3 + \Delta_1\omega = \Delta_5\omega(\omega^2 - \omega_{r4}^2)(\omega^2 - \omega_{r5}^2)$. An antiresonance is defined by the amplitude-frequency characteristic corresponding to the frequencies at which the numerator of dependence (20) is minimum or is equal to zero. In this case, the zero of the amplitude-frequency characteristic points to an evident resonance and its minimum points to a restricted resonance.

A restricted antiresonance arises at a frequency at which the numerator of dependence (20) is minimum. When one of the frequencies (real numbers) ω_{*1i} or ω_{*2i} is equal to any frequency from the group $\{0, \omega_{*3i}, \omega_{*4i}\}$, the antiresonance is strong and $\left|X_i(I\omega)\right| = 0$.

A resonance arises when the denominator in formula (20) is close to zero. If only one of the frequencies (real numbers) $\{\omega_{r1}, \omega_{r2}, \omega_{r3}\}$ is close to $\{0, \omega_{r4}, \omega_{r5}\}$, the resonance is infinite: $\left|X_i(I\omega)\right| \to \infty$; otherwise it is restricted.

The system of vibration isolation, shown in Fig. 1, can be easily transformed into any other system [3] by removal of unnecessary components. For a rigid base, $m_0 \to \infty$; in this case, $b_{00} = 0$, $b_{01} = 0$, $b_{02} = 0$, $b_{03} = 0$, $c_{00} = 0$, $c_{01} = 0$, $c_{02} = 0$, and $c_{03} = 0$. For the end m_2 free at the top (Fig. 1), the mass $m_3 = 0$, $c_2 = 0$, $b_2 = 0$, $c_3 =$

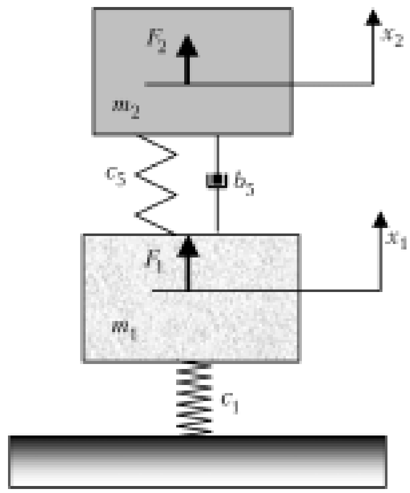

Fig. 2. Dynamic system of vibration isolation with a viscous friction.

0, $b_3 = 0$, $c_6 = 0$, and $b_6 = 0$. If the masses m_1 and m_2 are rigidly connected, $M = m_1 + m_2$, $c_5 = 0$, $b_5 = 0$, and $x(t) = x_1(t) + x_2(t)$.

For example for the dynamic system of vibration isolation with a viscous friction, presented in Fig. 2, $b_{00} = b_{01} = b_{02} = b_{03} = b_{10} = 0$, $b_{11} = 20$, $b_{12} = 20$, $b_{13} = b_{20} = 0$, $b_{21} = 10$, $b_{22} = 10$, $b_{23} = b_{30} = b_{31} = b_{32} = b_{33} = 0$, $c_{00} = c_{01} = c_{02} = c_{03} = c_{10} = 0$, $c_{11} = 4500$, $c_{12} = 800$, $c_{13} = c_{20} = 0$, $c_{21} = 1000$, $c_{22} = 1000$, $c_{23} = c_{30} = c_{31} = c_{32} = c_{33} = 0$, $f_1(t) = a\cos(\omega t)$, $f_2(t) = 0$, $f_3(t) = 0$, and $a = 1$.

By the form of the characteristic polynomial of the problem

$$Q(p) = a_0 + a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4 \tag{21}$$

one can judge the vibration resistance of a mechanical system as well as its vibration-resistance margin and quality or see that this system does not satisfy the simplest necessary conditions [4]. Such conditions of vibration resistance and quality of linear radio-engineering systems were formulated by V. S. Voronov in the 1960s of the last century [5]. Undoubtedly, these criteria can be extended to linear dynamic systems, the mathematical representation of which is analogous to that of radio-engineering systems with transfer functions in which the denominators represent characteristic polynomials.

In the example considered, where $a_0 = 3.7\cdot10^6$, $a_1 = 37{,}000.0$, $a_2 = 5500.0$, $a_3 = 30.0$, and $a_4 = 1.0$, the conditions necessary for the vibration resistance of the system: $a_0/a_2 < a_1/a_3 < a_2/a_4$ and the conditions sufficient for its stability: $W_k = a_k a_{k+1}/a_{k-1}a_{k+2} > 1$, are fulfilled. A mechanical object used in practice should have any vibration-resistance and quality margins determined from simple relations between the parameters of polynomial (21): $W_k > 3$ and $\Omega_k = a_k^2/a_{k-1}a_{k+2} > \sqrt{3}$ [5]. In the example considered, $W_1 = 1.8 < 3$ and $\Omega_1 = 0.067 > \sqrt{3}$, $\Omega_2 = 27.3 < \sqrt{3}$, and $\Omega_3 = 0.16 < \sqrt{3}$. The vibrations of the vibration-isolation system shown in Fig. 2 are stable; however, this system has no vibration-resistance and quality margins (Fig. 3).

We now consider a high-quality system. Let us assume that $b_{11} = 200$, $b_{12} = 20$, $b_{21} = 100$, $b_{22} = 100$, $c_{11} = 100$, $c_{12} = 16$, $c_{21} = 80$, and $c_{22} = 120$. In this case, the characteristic polynomial has the form $Q(p) = 10{,}720 + 30{,}800.0p + 18{,}220.0p^2 + 300.0p^3 + p^4$ and all the conditions of vibration resistance are fulfilled, i.e., the necessary conditions: $0.6 < 102.7 < 18{,}220.0$, the sufficient conditions: $174.5 > 1$ and $17.5 > 1$, the conditions of vibration resistance with a margin: $174.5 > 3$ and $177.5 > 3$, and the quality conditions: $4.9 > \sqrt{3}$, $36.0 > \sqrt{3}$, and $5.0 > \sqrt{3}$ (Fig. 4). Since $\Omega_i \geq 4$ ($i = \overline{1, 3}$), all the roots of the characteristic polynomial (21) will be negative and real and the natural vibrations will be damped.

The physical model of such a system involves an additional rigid element c_4 (Fig. 2) positioned between the masses m_1 and the base m_0.

Let us verify if the conditions of parallel and series connection of elements (Fig. 1) are fulfilled [1] in the mathematical model (5)–(7). It is known that in the case of series connection of springs, e.g., of c_2 and c_4, the total

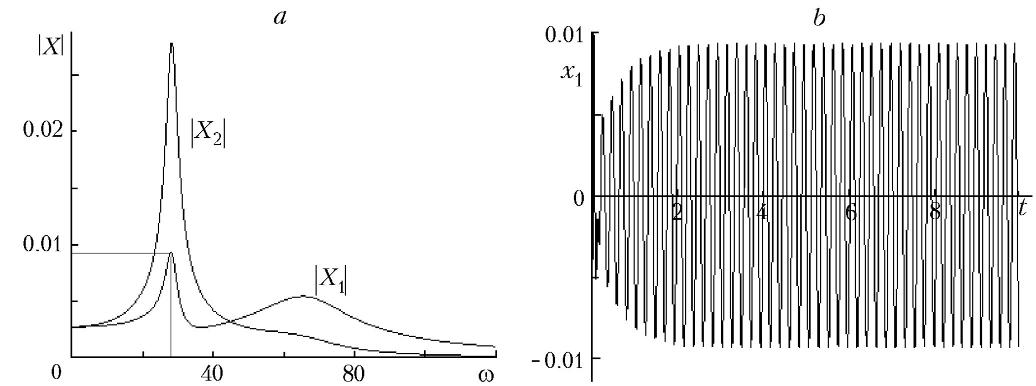

Fig. 3. Amplitude-frequency characteristics of a dynamic spring vibration damper with a viscous friction and a resonance of vibrations $x_1(t)$ at a frequency ω = 28.0 rad/sec (a); time dependence of the amplitude of vibrations $x_1(t)$ at $f_1(t) = \cos(28t)$ (b).

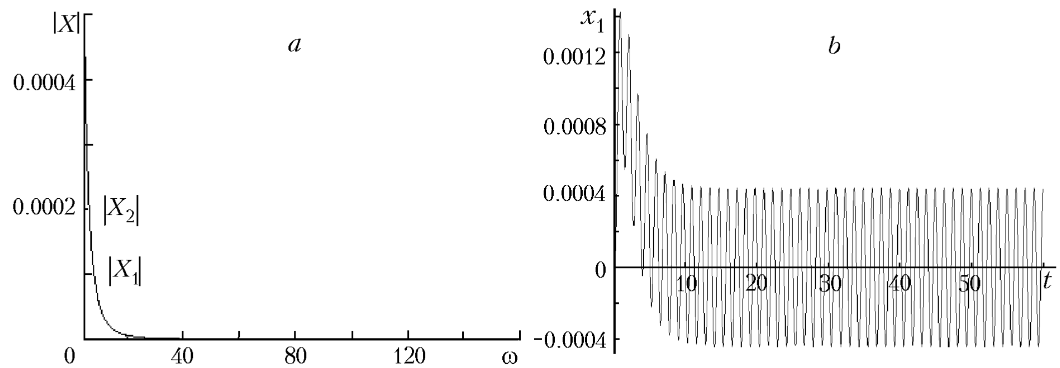

Fig. 4. Amplitude-frequency characteristics $|X_1(\omega)|$ and $|X_2(\omega)|$ that are practically coincident (a); forced vibrations of the mass m_1 with an amplitude $x_1(t)$ at $f_1(t) = \cos(5t)$ (b).

rigidity is equal to the ratio between unity and the sum of their reciprocals: $1/(1/c_2 + 1/c_4)$. Let us assume that all the damping coefficients $b_i = 0$ $(i = \overline{1, 6})$, the masses $m_1 = m_2 = 0$, the rigidities $c_1 = c_3 = c_5 = c_6 = 0$, and the external loads $\mathcal{F}_1 = \mathcal{F}_2 = 0$ are absent in the system presented in Fig. 1. In this case, the system of equations (5)–(7) takes the form $0 = -(c_2 + c_4)x_2 + c_2x_3$, $m_3\ddot{x}_3 = c_2(x_2 - x_3) + m_3\mathcal{F}_3$. From the first relation we obtain $x_2 = c_2/(c_2 + c_4)$ and, substituting this expression it into the second relation, obtain $m_3\dot{x}_3 + c_2c_4/(c_2 - c_4)x_3 = m_3\mathcal{F}_3$. Consequently, the total rigidity of the mechanical system is equal to $c_2c_4/(c_2 + c_4)$.

In the case of parallel connection of springs, the rigidities are added. Let us assume that the springs c_3 and c_6 are connected in parallel between the mass m_3 and the base m_0 (Fig. 1), the damping elements $b_i = 0$ $(i = \overline{1, 6})$ are absent, the mass $m_2 = 0$, $m_1 \to \infty$ is connected to the base, the rigidities $c_1 = c_2 = c_4 = c_5 = 0$, and the external loads $\mathcal{F}_1 = \mathcal{F}_2 = 0$. In this case, the system of equations (5)–(7) is simplified to one equation: $m_3\ddot{x}_3 + (c_3 + c_6)x_3 = m_3\mathcal{F}_3$. The total rigidity of this mechanical system is equal to the sum $c_3 + c_6$.

The aforesaid allows the conclusion that the procedure of parallel and series connection of physical elements, used in the theory of chain dynamic systems or in the impedance theory [1], is performed automatically with mathematical accuracy in the method presented and is involved in the dynamics problem (1)–(4).

The method developed for investigating complex dynamic mechanisms with the use of a sequence of the differential operators L_i $(i = \overline{1, 3})$ makes it possible to exclude cumbersome mathematical calculations and estimate the quality of these mechanisms and the stability of their vibrations by the known coefficients of the general mathematical model (1)–(4). The approach proposed conforms with the method of integral Laplace transformations, the method of variation of stationary amplitudes, the method of dynamic systems, and the impedance method. Its merit is the possibility of determining the desired vibrations of masses and their by solving independent linear inhomogeneous differential equations. This provides an additional condition for verification of numerical calculations it because allows one to conveniently represent the vibrations of masses in the phase plane, characterize the form of the phase trajectories, the type of singular points, and determine the equilibrium position of a mechanical vibratory system.

NOTATION

A, amplitude of vibrations of the external force F, N; a, amplitude of vibrations of the external force related to the mass m_0, N/kg; a_0, a_1, a_2, a_3, a_4, additional constants; c_i, coefficient of elasticity of springs, kg·sec^{-2}; b_i, coefficient of damping elements, kg·sec^{-1}; F, F_1, F_2, F_3, external forces, N; $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_3$, vibration accelerations related to corresponding masses, m/sec^{-2}; L_1, L_2, L_3, d_{12}, d_{13}, d_{21}, d_{23}, d_{31}, d_{32}, differential operators of the second and first orders relative to the time parameter t; m_0, m_1, m_2, m_3, masses of elements of a vibration damper, kg; ω, frequency of vibrations of the external force F, rad·sec^{-1}; ω_{*1i}, ω_{*2i}, ω_{*3i}, ω_{*4i}, antiresonance frequencies; ω_{r1}, ω_{r2}, ω_{r3}, ω_{r4}, and ω_{r5}, resonance frequencies. Subscript: r, resonance.

REFERENCES

1. C. M. Harris and Ch. E. Crede, *Shock and Vibration Handbook* [Russian translation], Sudostroenie, Leningrad (1980).
2. N. A. Dokukova and P. N. Konon, Equivalence of the impedance method and the method of amplitude-frequency characteristics for investigations of vibrations in hydraulically powered supports, *Inzh.-Fiz. Zh.*, **76**, No. 6, 174–177 (2003).
3. N. I. Levitskii, *Vibrations in Mechanisms* [in Russian], Nauka, Moscow (1988).
4. E. von Kamke, *Differentialgleichungen Lösungsmethoden und Lösung* [Russian translation], Nauka, Moscow (1971).
5. L. M. Skvortsov and B. T. Fedosov, About V. S. Voronov algebraic criteria of stability and quality of linear systems //http://model.exponenta.ru/bt/bt_00118.html.
6. N. A. Dokukova and M. D. Martynenko, Method of linearization and its application to analysis of linear vibrational systems, *Vestsi Nats. Akad. Navuk Belarusi*, *Ser. Fiz.-Tékhn. Navuk*, No. 3, 108–111 (1999).
7. M. D. Martynenko, N. A. Dokukova, and L. I. Boiko, Vibrational processes in mechanical systems, *Inzh.-Fiz. Zh.*, **72**, No. 3, 491–494 (1999).